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Robust Slope Estimators for Simple Linear Regression Model

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ABSTRACT
Estimators based on median and Hodges-Lehmann (HL) techniques for estimating the slope parameter of simple linear regression (SLR) are robust in nature. In this paper, we develop slope estimators based on median and HL estimator of slopes obtained from quasi ranges of predictor variables. The mean and variance of the proposed estimators are obtained when errors have various symmetric distributions. Their performance is evaluated and they are compared with their competitors. Their generalized forms are discussed for the situations of multiple responses. Also, fitting of SLR model using the proposed estimators is illustrated with a data set.

KEYWORDS
Median based estimator, Quasi range, Robust, Simple linear regression, Slope parameter.

1. Introduction

The simple linear regression model is given by

$$y_j = \alpha + \beta x_j + e_j, \qquad j = 1, \ldots, n, \tag{1}$$

where y_j is response variable, x_j is predictor variable and e_j is a random error. We assume that, $e_j s$ are independent and identically distributed (iid) from absolutely continuous symmetric distribution with distribution function $F(\cdot)$ having mean zero and variance σ^2. Here, α is intercept parameter and β is slope parameter to be estimated from the data to explore the linear relation between x_j and y_j. The slope parameter β represents the change in mean of distribution of y for unit change in x.

Legendre [13] and Gauss [8] proposed least square estimator to β by minimizing error sum of squares, $\sum_{j=1}^{n} e_j^2$. Bose [6] developed three estimators based on methods of successive differences, differences at half range and range of predictor variables, when $x_j s$ have equal distance, in particular unit distance. Nair and Shrivastava [15] generalized this procedure by considering optimum number of groups while dividing error equations.

Suppose (x_j, y_j), $j = 1, \cdots, n$ are pair of observations and are arranged in increasing order with respect to x_j, $x_{(j)}$ is the j^{th} order statistic and y_j^* is the corresponding

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y value, then for $n = 2m$, the i^{th} half range is given by

$$h_i = \left(x_{(m+i)} - x_{(i)}\right), \quad i = 1,\ 2,\ \cdots, m \tag{2}$$

and i^{th} quasi range is given by

$$q_i = x_{(2m-i)} - x_{(i+1)}\ , \qquad i = 1,\ 2,\ \cdots, m-1. \tag{3}$$

We observe that, $q_0 = x_{(2m)} - x_{(1)}$ is the range of n observations.

Theil [21] suggested a complete method to obtain estimators based on median of all possible slopes from pairwise observations, $1 \le i < j \le n$ and an incomplete method to obtain estimator, $\widehat{\beta}_{TM}$ based on median of slopes obtained using differences at half range given by

$$b_i = \frac{y^*_{m+i} - y^*_i}{h_i}\ , \quad i = 1,\ 2,\ \cdots, m. \tag{4}$$

Sen [19] extended complete method to the case where not all $x_{(j)}$ need to be distinct, which is popularly known as Theil-Sen estimator. Scholz [18] and Sievers [20] generalized the Theil-Sen estimator using weights to the slopes of pairwise observations. Gore and Rao [9] and Rao [16] used Theil's incomplete method to regression model considering multiple y_j observations at each value of x_j under the assumption of equidistant $x_j s$. They extended their study to HL estimator due Hodges and Lehmann [10], which is median of Walsh averages. Also, Rao [16] discussed about $\widehat{\beta}_{TA}$ which is an estimator of β based on slopes given in (4).

Hussain and Sprent [11] observed that, Kildea [12] obtained median of the subset of slopes of $(n-1)$ elements given by

$$b_i = \tfrac{y^*_{n-i+1} - y^*_i}{x_{(n-i+1)} - x_{(i)}} \text{ for } i \le \tfrac{n}{2} \text{ and } b_{i+1} = \tfrac{y^*_{n-i+1} - y^*_i}{x_{(n-i+1)} - x_{(i)}} \text{ for } i \le \tfrac{n-1}{2}$$

when all $x_{(j)} s$ are distinct. They suggested alternative choice of subset given by $b_i = \frac{y^*_{i+m} - y^*_i}{x_{(i+m)} - x_{(i)}}$ and $b_{i+1} = \frac{y^*_{i+m+1} - y^*_i}{x_{(i+m+1)} - x_{(i)}}$ for $m = \frac{n}{2}$ if n is even and $m = \frac{n-1}{2}$ if n is odd. It is observed that, the subsets chosen by Kildea [12] and Hussain and Sprent [11] are dependent. Liu and Preve [14] developed an estimator on the assumption that predictor variable is stochastic and error has distribution either normal mixture or symmetric stable which is symmetric and heavy tailed. Cliff and Billy [7] proposed a method based on the average of slopes of successive differences. Bhat and Bijjargi [2] and [3] proposed $\widehat{\beta}_Q$ and $\widehat{\beta}_W$ based respectively on average and weighted average of quasi ranges. Also, Bhat and Bijjargi [4] and [5] dealt with classes of estimators dependent on average of slopes based on quasi ranges respectively given by $\widehat{\beta}_{2C_k}$ utilizing constant weights and $\widehat{\beta}_{2D_k}$ utilizing varying weights based on quasi ranges. They found that, $\widehat{\beta}_{2C_2}$ and $\widehat{\beta}_{2D_2}$ were optimal members of these classes of estimators.

We propose robust estimators of β based on quasi ranges in section 2. The mean and variance of the proposed estimators are derived in section 3. Their performance is discussed in section 4. In section 5, the proposed estimators are generalized to the situations of occurrence of multiple responses. They are illustrated through an example in section 6 and section 7 contains conclusions. The computed tables supporting our study are given in appendix.

2. Proposed median based estimators

The median and HL estimators are robust estimators of location. They are resistant to outliers. When observations are arranged in ascending or descending order, median is the middle observation that divides the data into two equal parts. Suppose, t_1, t_2,$\cdots$,t_n be random sample of size n from absolutely continuous distribution with distribution function $F(t)$, then the median of t say t_M has limiting distribution,

$$N\left(F^{-1}\left(\frac{1}{2}\right),\left(4nf^2\left(F^{-1}\left(\frac{1}{2}\right)\right)\right)^{-1}\right), \tag{5}$$

where f is the probability density function evaluated at point $F^{-1}\left(\frac{1}{2}\right)$. The HL estimator is defined as median of $\frac{n(n+1)}{2}$ Walsh averages, where, $w_{ij} = \frac{(t_i+t_j)}{2}$, $1 \leq i \leq j \leq n$ are Walsh averages. Suppose t_is are symmetric around θ, then HL estimator of t, say t_{HL} , has distribution,

$$N\left(\theta,\left(12n\,\overline{f}^{\,2}\right)^{-1}\right) \quad \text{where, } \overline{f} = \int f^2(t)\ dt. \tag{6}$$

Here, we propose estimators based on median and HL type estimator of independent slopes, b_i obtained using quasi ranges.

$$\text{Suppose,} \quad b_i = \frac{y^*_{m+i} - y^*_{m-i+1}}{q_{m-i}} \quad \text{for} \quad i = 1,\ \cdots, m. \tag{7}$$

The estimator based on median of b_is is given by

$$\widehat{\beta}_M = med\left(b_i\right), \quad 1 \leq i \leq m \tag{8}$$

and the adaptive estimator which is HL type estimator based on the median of Walsh averages of b_is is given by

$$\widehat{\beta}_A = med\left(\frac{b_i + b_{i'}}{2}\right), \quad 1 \leq i \leq i' \leq m. \tag{9}$$

3. Mean and Variance of Proposed estimators

In this section, the mean and variance of proposed estimators are obtained. From equation (7),

$$\begin{aligned} b_i &= \beta + \frac{e_{m+i} - e_{m-i+1}}{q_{m-i}} \\ &= \beta + u_i, \end{aligned}$$

where, $u_i = \frac{e_{m+i}-e_{m-i+1}}{q_{m-i}}$ is a continuous random variable symmetric around zero. Hence, $E\left(b_i\right) = \beta$, $i = 1,\ 2,\ \cdots m$.

The mean of $\widehat{\beta}_M$ is given by

$$E(\widehat{\beta}_M) = E(med\,(b_i))$$

$$= \beta + E(med(u_i)).$$

$$E(\widehat{\beta}_M) = \beta, \quad \text{since } E\,(med\,(u_i)) = 0. \tag{10}$$

Hence, $\widehat{\beta}_M$ is an unbiased estimator of β. As sample median is consistent estimator of population median, $\widehat{\beta}_M$ is consistent estimator of β.

On similar lines, we notice that, $\widehat{\beta}_A$ is also, an unbiased and consistent estimator of β. From equation (5) and equation (6), we observe that,

$$\widehat{\beta}_M \;\sim\; N\left(\beta, \frac{1}{4mf^2(\beta)}\right) \tag{11}$$

$$\text{and} \qquad \widehat{\beta}_A \;\sim\; N\left(\beta, \frac{1}{12m\overline{f}^2}\right). \tag{12}$$

The distribution of b_i depends on the distribution of u_i and that of u_i depends on difference of $e_j s$. Hence, in order to obtain the variance of $\widehat{\beta}_M$ and $\widehat{\beta}_A$, we consider the distribution of e_j to be uniform(U), normal(N), Laplace(L) and Cauchy(C) distribution. The distribution of e_j other than Cauchy distribution is modified in such a way that its mean is zero and variance is σ^2. The scale parameter of Cauchy distribution is taken as λ.

Suppose we consider that, error has uniform distribution, that is, $e_j \sim U(-\sqrt{3}\sigma, \sqrt{3}\sigma)$, then $u_i \sim Tr\left(-\frac{2\sqrt{3}\sigma}{C_i}, \frac{2\sqrt{3}\sigma}{C_i}, 0\right)$ where, $C_i = q_{m-i},\ i = 1,\ 2,\ \cdots, m$ and $Tr(p,\ q, r)$ is triangular distribution with $p,\ q,\ r$ given respectively by minimum, maximum and most likely value of u_i. The probability density function of u_i is given by

$$f\,(u_i) = \begin{cases} \frac{C_i\left(C_i u_i + 2\sqrt{3}\sigma\right)}{12\sigma^2} & ;\ -\frac{2\sqrt{3}\sigma}{C_i} < b_i \le 0 \\ \frac{C_i\left(2\sqrt{3}\sigma - C_i u_i\right)}{12\sigma^2} & ;\ 0 < b_i \le \frac{2\sqrt{3}\sigma}{C_i} \end{cases} \tag{13}$$

Therefore, $b_i \sim Tr\left(\beta - \frac{2\sqrt{3}\sigma}{C_i}, \beta + \frac{2\sqrt{3}\sigma}{C_i}, \beta\right)$ is symmetric around β. The density of b_i is given by

$$f\,(b_i) = \begin{cases} \frac{C_i^2\left(b_i - \beta + \frac{2\sqrt{3}\sigma}{C_i}\right)}{12\sigma^2} & ;\ \beta - \frac{2\sqrt{3}\sigma}{C_i} < b_i \le \beta \\ \frac{C_i^2\left(-b_i + \beta + \frac{2\sqrt{3}\sigma}{C_i}\right)}{12\sigma^2} & ; \beta < b_i \le \beta + \frac{2\sqrt{3}\sigma}{C_i} \end{cases} \tag{14}$$

From (14) and (11) we get, $f^2(\beta) = \frac{C_M^2}{12\sigma^2}$ and $V\left(\widehat{\beta}_M\right) = \frac{3\sigma^2}{mC_M^2}$ respectively, where, C_M is q_{m-i} of the median of $b_i s$.

To obtain the $V(\widehat{\beta}_A)$, when $e_j \sim U(-\sqrt{3}\sigma, \sqrt{3}\sigma)$, $\overline{f} = \int_{\beta-\frac{2\sqrt{3}\sigma}{C_i}}^{\beta+\frac{2\sqrt{3}\sigma}{C_i}} f^2(b_i)\ db_i = \frac{C_A}{3\sqrt{3}\sigma}$. Hence, $V\left(\widehat{\beta}_A\right) = \frac{1}{12m\overline{f}^2} = \frac{9\sigma^2}{4mC_A^2}$, where, C_A is q_{m-i} of HL type estimator of $b_i s$. On similar lines, $V\left(\widehat{\beta}_M\right)$ and $V\left(\widehat{\beta}_A\right)$ are obtained for various distributions and furnished in Exhibit 1.

Exhibit 1.: Variance of $\widehat{\beta}_M$ and $\widehat{\beta}_A$ for various distributions

Distribution of e_j	$f(\beta)$	$V\left(\widehat{\beta}_M\right)$	$\overline{f}$	$V\left(\widehat{\beta}_A\right)$
$U(-\sqrt{3}\sigma, \sqrt{3}\sigma)$	$\frac{\sqrt{3}C_M}{6\sigma}$	$\frac{3\sigma^2}{mC_M^2}$	$\frac{C_A}{3\sqrt{3}\sigma}$	$\frac{9\sigma^2}{4mC_A^2}$
$N(0, \sigma^2)$	$\frac{C_M^2}{2\sqrt{2\pi\sigma^2}}$	$\frac{\pi\sigma^2}{mC_M^2}$	$\frac{C_A}{2\sqrt{2\pi}\sigma}$	$\frac{2\pi\sigma^2}{3mC_A^2}$
$L\left(0, \frac{\sqrt{2}}{\sigma}\right)$	$\frac{\sigma C_M}{4\sqrt{2}}$	$\frac{8}{m\sigma^2 C_M^2}$	$\frac{5\sigma C_A}{32\sqrt{2}}$	$\frac{512}{75m\sigma^2 C_A^2}$
$C(0, \lambda)$	$\frac{C_M}{2\pi\lambda}$	$\frac{\pi^2\lambda^2}{mC_M^2}$	$\frac{C_A}{4\lambda\pi}$	$\frac{4\lambda^2\pi^2}{3mC_A^2}$

4. Performance of $\widehat{\beta}_M$ and $\widehat{\beta}_A$

In this section, we evaluate the performance of $\widehat{\beta}_M$, $\widehat{\beta}_A$, and compare them with their competitors, viz. $\widehat{\beta}_{LS}$, $\widehat{\beta}_Q$, $\widehat{\beta}_W$, $\widehat{\beta}_{2C_2}$, $\widehat{\beta}_{2D_2}$, $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$ given in Exhibit 2.

Exhibit 2.: Some competitors and their variance

Name of the estimators	Estimators	Variance
Least square estimator	$\widehat{\beta}_{LS} = \frac{\sum_{j=1}^{n}(x_j-\overline{x})(y_j-\overline{y})}{\sum_{j=1}^{n}(x_j-\overline{x})^2}$	$\frac{\sigma^2}{\sum_{j=1}^{n}(x_j-\overline{x})^2}$
Estimator due to Bhat and Bijjargi [2]	$\widehat{\beta}_Q = \frac{\sum_{i=1}^{m}\left(y_{m+i}^*-y_{m-(i-1)}^*\right)}{\sum_{i=1}^{m} q_{m-i}}$	$\frac{n\sigma^2}{\left(\sum_{i=1}^{m} q_{m-i}\right)^2}$
Estimator due to Bhat and Bijjargi [3]	$\widehat{\beta}_W = \frac{\sum_{i=1}^{m} \frac{i\left(y_{m+i}^*-y_{m-(i-1)}^*\right)}{\sum_{i=1}^{m} i}}{\sum_{i=1}^{m} \frac{i(q_{m-i})}{\sum_{i=1}^{m} i}}$	$\frac{2\sigma^2\sum_{i=1}^{m} i^2}{\left(\sum_{i=1}^{m} i(q_{m-i})\right)^2}$
Estimator due to Bhat and Bijjargi [4]	$\widehat{\beta}_{2C_2} = \frac{\sum_{i=1}^{m} i^2 \frac{\left(y_{m+i}^* - y_{m-(i-1)}^*\right)}{q_{m-i}}}{\sum_{i=1}^{m} i^2}$	$\frac{2\sigma^2}{\left(\sum_{i=1}^{m} i^2\right)^2}\sum_{i=1}^{m}\frac{i^4}{q_{m-i}^2}$

Estimator due to Bijjargi and Bhat [5]	$\widehat{\beta}_{2D_2} = \frac{\sum_{i=1}^{m} q_{m-i}\left(y^*_{m+i} - y^*_{m-(i-1)}\right)}{\sum_{i=1}^{m} q^2_{m-i}}$	$\frac{2\sigma^2}{\sum_{i=1}^{m} q^2_{m-i}}$
Theil's incomplete estimator using median due to Theil [21]	$\widehat{\beta}_{TM} = \text{med}\left(\frac{y^*_{m+i} - y^*_i}{h_i}\right)$	$\frac{3\sigma^2}{mC^2_{TM}}$ under $U(-\sqrt{3}\sigma, \sqrt{3}\sigma)$ $\frac{\pi\sigma^2}{mC^2_{TM}}$ under $N(0, \sigma^2)$ $\frac{8}{m\sigma^2 C^2_{TM}}$ under $L\left(0, \frac{\sqrt{2}}{\sigma}\right)$ $\frac{\pi^2\lambda^2}{mC^2_{TM}}$ under $C(0, \lambda)$
Theil's incomplete estimator using HL estimator due to Rao [16]	$\widehat{\beta}_{TA} = HL\left(\frac{y^*_{m+i} - y^*_i}{h_i}\right)$	$\frac{9\sigma^2}{4mC^2_{TA}}$ under $U(-\sqrt{3}\sigma, \sqrt{3}\sigma)$ $\frac{2\pi\sigma^2}{3mC^2_{TA}}$ under $N(0, \sigma^2)$ $\frac{512}{75m\sigma^2 C^2_{TA}}$ under $L\left(0, \frac{\sqrt{2}}{\sigma}\right)$ $\frac{4\lambda^2\pi^2}{3mC^2_{TA}}$ under $C(0, \lambda)$

where $\overline{x} = \frac{\sum_{j=1}^{n} x_j}{n}$, $\overline{y} = \frac{\sum_{j=1}^{n} y_j}{n}$, C_{TM} and C_{TA} are h_i of $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$ respectively.

From the variance of proposed estimators under different distributions given in Exhibit 1, for $\sigma = 1$, $\lambda = 0.2605$ (Arnold [1]) and $C_M = C_A = 1$, the variance of $\widehat{\beta}_M$ and $\widehat{\beta}_A$ for different values of n are computed and given in Table 1 in appendix and plotted in Figure 1.

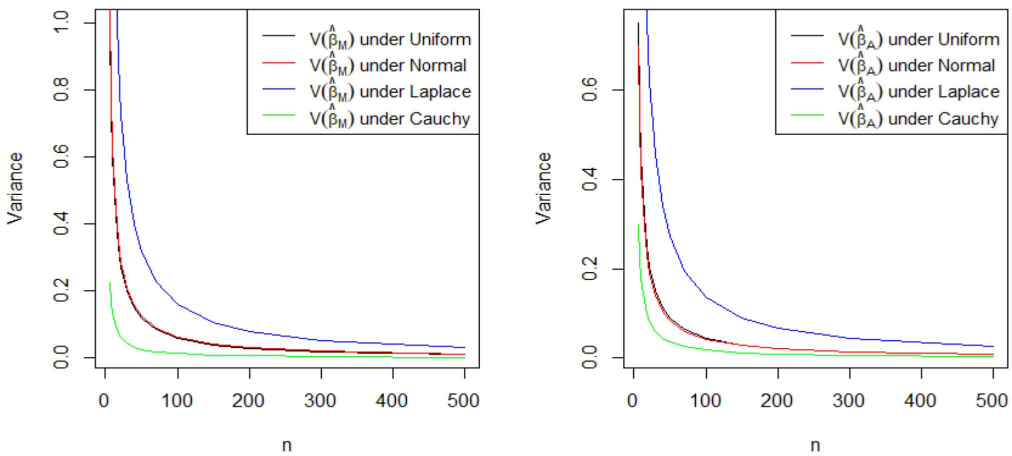

Figure 1.: $V(\widehat{\beta}_M)$ and $V(\widehat{\beta}_A)$ under various distributions

From Table 1 and Figure 1, it is observed that, the $V(\widehat{\beta}_M)$ and $V(\widehat{\beta}_A)$ decrease as n increases for all the distributions under consideration. Also, $V(\widehat{\beta}_M) > V(\widehat{\beta}_A)$ for uniform, normal and Laplace distributions whereas $V(\widehat{\beta}_M) < V(\widehat{\beta}_A)$ for Cauchy distribution.

The relative efficiency (RE) that establishes relative performance of estimators in terms of precision among $\widehat{\beta}_A$ and $\widehat{\beta}_M$ are obtained when $C_M = C_A$. It is given by

$$RE\left(\widehat{\beta}_A, \widehat{\beta}_M\right) = \frac{V(\widehat{\beta}_M)}{V(\widehat{\beta}_A)} \tag{15}$$

and is computed for different distributions in Exhibit 3.

Exhibit 3.: RE among proposed estimators

Distribution of e_j	Uniform	Normal	Laplace	Cauchy
$RE(\widehat{\beta}_A, \widehat{\beta}_M)$	1.33	1.50	1.17	0.75

From Exhibit 3, it is observed that, under error having uniform, normal and Laplace distributions, $\widehat{\beta}_A$ performs better than $\widehat{\beta}_M$, whereas, $\widehat{\beta}_M$ performs better than $\widehat{\beta}_A$ under Cauchy distribution.

To compare the $V(\widehat{\beta}_M)$ and $V(\widehat{\beta}_A)$ with their competitors given in Exhibit 2, the random errors of size n are generated 10,000 times from symmetric distributions under consideration with mean zero and various values of σ, λ. The y_js are evaluated using generated e_js, considering $\alpha = 2$, $\beta = 3$ and generating a random sample of x_js. The computations are carried out for obtaining variances of $\widehat{\beta}_M$, $\widehat{\beta}_A$, $\widehat{\beta}_{TM}$, $\widehat{\beta}_{TA}$ under various symmetric distributions and $\widehat{\beta}_{LS}$, $\widehat{\beta}_Q$, $\widehat{\beta}_W$, $\widehat{\beta}_{2C_2}$, $\widehat{\beta}_{2D_2}$ under normal distribution. The computed values for various n, σ, λ are provided in Table 2, Table 3 and are given in appendix. Since, there exist 10,000 values of $V(\cdot)$ for each estimator, the median of $V(\cdot)$ is taken as an estimator of $V(\cdot)$ given by $\widehat{V}(\cdot)$.

From Table 2, it is observed that, both $\widehat{\beta}_A$ and $\widehat{\beta}_M$ outperform $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$. Under uniform and normal error distributions, $\widehat{\beta}_A$ performs better than $\widehat{\beta}_M$, $\widehat{\beta}_{TM}$, $\widehat{\beta}_{TA}$ and $\widehat{\beta}_M$ performs better than $\widehat{\beta}_{TM}$, $\widehat{\beta}_{TA}$. Under Laplace error distribution, for small n, $\widehat{\beta}_M$ outperforms $\widehat{\beta}_A$, $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$. Under Cauchy distribution, $\widehat{\beta}_M$ performs better than $\widehat{\beta}_A$, $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$. For increasing values of n and σ, the variance of $\widehat{\beta}_M$, $\widehat{\beta}_A$, $\widehat{\beta}_{TM}$, $\widehat{\beta}_{TA}$ decrease for Laplace error distribution whereas, variance increases for other distributions.

From Tables 2 and 3, it is seen that, for all the distributions, variance of all the estimators under consideration decrease with increasing values of n. Under uniform and normal error distributions, $\widehat{\beta}_A$ outperforms $\widehat{\beta}_{LS}$, $\widehat{\beta}_Q$, $\widehat{\beta}_W$, $\widehat{\beta}_{2C_2}$, $\widehat{\beta}_{2D_2}$ for $n > 30$ and $n \geq 20$ respectively. Under Laplace distribution, for $\sigma \geq 5$, $\widehat{\beta}_M$ and $\widehat{\beta}_A$ outperforms $\widehat{\beta}_{LS}$, $\widehat{\beta}_Q$, $\widehat{\beta}_W$, $\widehat{\beta}_{2C_2}$ and $\widehat{\beta}_{2D_2}$.

Also, it is noticed from the simulation study that, when predictor variables are equidistant with distance being d, say $d = 1, 10, 50, 100$ and errors follow various distributions, $\widehat{\beta}_M$ and $\widehat{\beta}_A$ outperform $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$. For increasing values of n and d, $\widehat{\beta}_M$ and $\widehat{\beta}_A$ perform better than $\widehat{\beta}_{LS}$, $\widehat{\beta}_Q$, $\widehat{\beta}_W$, $\widehat{\beta}_{2C_2}$ and $\widehat{\beta}_{2D_2}$.

5. Generalization of proposed estimators for multiple responses

In this section, we give generalized versions of proposed estimators. The situation may arise where multiple y_js are observed for single x_j observation. For example, in medical research, consider x_j as the dosage of a medicine given to a patient and y_j as the response of the patient measured at different time points (e.g., blood pressure, heart rate, or symptom severity). The model would assess how changes in dosage affect the observed responses over time. In manufacturing processes, x_j could represent the temperature or pressure in a production process, while y_js could represent the quality or strength of the product measured at different stages of production. This model could help identify optimal process conditions for desired product outcomes. In stock market analysis, x_j could represent factors such as company earnings, dividend yields, or market indices, while y_js could represent stock prices observed at different time points. This model can be used to analyze how changes in fundamental factors affect stock prices over time. In such situations, the regression model given in equation (1) is not directly applicable and hence can be generalized as

$$y_{jk} = \alpha + \beta x_j + e_{jk}, \quad j = 1, \ \ldots, \ n; \ k = 1, \ \ldots, \ p. \tag{16}$$

That is,

$$\begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{np} \end{pmatrix} = \alpha + \beta \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} e_{11} & e_{12} & \cdots & e_{1p} \\ e_{21} & e_{22} & \cdots & e_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ e_{n1} & e_{n2} & \cdots & e_{np} \end{pmatrix},$$

where, y_{jk} denotes responses for j^{th} observation at k^{th} instance. The e_{jk}s are errors associated with y_{jk}s and are symmetric around zero with variance σ^2. Let the data on (y_{jk}, x_j) be arranged in increasing order with respect to x_j and y^*_{jk} be y_{jk} observations corresponding to $x^{th}_{(j)}$ order statistics. Also, y^*_{jk} are random and arranged in order in which they have occurred. On similar arguments for estimators based on half ranges proposed by Rao [16], we get p independent slopes based on quasi ranges as below.

quasi ranges (q_i)	Slopes obtained from q_i			
q_0	$\frac{y^*_{n1}-y^*_{11}}{q_0}$	$\frac{y^*_{n2}-y^*_{12}}{q_0}$	$\cdots$	$\frac{y^*_{np}-y^*_{1p}}{q_0}$
q_1	$\frac{y^*_{(n-1)1}-y^*_{21}}{q_1}$	$\frac{y^*_{(n-1)2}-y^*_{22}}{q_1}$	$\cdots$	$\frac{y^*_{(n-1)p}-y^*_{2p}}{q_1}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
q_{m-1}	$\frac{y^*_{(m+1)1}-y^*_{m1}}{q_{m-1}}$	$\frac{y^*_{(m+1)2}-y^*_{m2}}{q_{m-1}}$	$\cdots$	$\frac{y^*_{(m+1)p}-y^*_{mp}}{q_{m-1}}$

Hence, we get $mp = N$ independent slopes, say b_s, $1 \le s \le N$. The generalized estimators are given by

$$\widehat{\beta}^*_M = med\left(b_s\right), 1 \le s \le N \tag{17}$$

$$\text{and} \qquad \widehat{\beta}^*_A = med\left(\frac{b_s + b_{s'}}{2}\right), 1 \le s \le s^{'} \le N. \tag{18}$$

The estimators $\widehat{\beta}^*_M$ and $\widehat{\beta}^*_A$ are also an unbiased and consistent estimators of β. The variances of $\widehat{\beta}^*_M$ and $\widehat{\beta}^*_A$ are obtained by replacing N in place of m in (13) and (14).

Additionally, there may be certain situations wherein it is useful to consider various functions of y_{jk} such as the maximum, minimum, average and median. For instance, in the example given on medical research, while monitoring blood pressure, identifying the maximum blood pressure recorded for a patient over a certain period indicates peak cardiovascular stress, while the minimum blood pressure might reflect the lowest physiological state experienced. Moreover, analyzing the average blood pressure provides insights into overall cardiovascular health and considering the median helps account for outliers or extreme values in the dataset. By incorporating various statistics of y_{jk}, healthcare professionals can gain a comprehensive understanding of patient responses. The model given in equation (16) can be modified as

$$g\left(y_{j\cdot}\right) = \alpha + \beta\ x_j + g(e_{j\cdot}) \tag{19}$$

where, $g(y_{j\cdot})$ is any function of $y_{j\cdot}$, such as mean, median, maximum or minimum and $g(e_{jk})$ is function of $e_{j\cdot}$s and are iid, symmetric around zero with constant variance σ^2_g. Here, the m slopes are given by

$$b'_i = \frac{g(y^*_{(n-i+1)\cdot}) - g(y^*_{i\cdot})}{q_{i-1}}, \qquad i = 1, 2, \cdots, m. \tag{20}$$

The proposed estimators under the model (19) are given by

$$\widehat{\beta}'_M = med\left(b'_i\right), \quad 1 \le i \le m \tag{21}$$

$$\text{and} \quad \widehat{\beta}'_A = med\left(\frac{b'_i + b'_{i'}}{2}\right), \quad 1 \le i \le i' \le m. \tag{22}$$

$$\text{Since,} \qquad b'_i = \frac{g(\alpha + \beta x_{(n-i+1)} + e_{(n-i+1)\cdot}) - g(\alpha + \beta x_{(i)} + e_{i\cdot})}{q_{i-1}}$$

$$= \beta + u'_i \tag{23}$$

where $u'_i = \frac{g\left(e_{(n-i+1)\cdot}\right) - g(e_{i\cdot})}{q_{i-1}}$ is symmetric around zero,

$$E(\widehat{\beta}'_M) = E\left(med\left(b'_i\right)\right)$$

$$= \beta + E\left(med(u'_i)\right).$$

$$\text{Hence,} \quad E(\widehat{\beta}'_M) = \beta, \qquad \text{since } E\left(med\left(u'_i\right)\right) = 0. \tag{24}$$

The variance of $\widehat{\beta}'_M$ is given by

$$V\left(\widehat{\beta}'_M\right) = \frac{1}{4mf'^2(\beta)} \tag{25}$$

where $f'(\cdot)$ is pdf of b'_i.

$$\text{Similarly,} \quad E(\widehat{\beta}'_A) = E\left(med\left(\frac{b'_i + b'_{i'}}{2}\right)\right)$$

$$= E\left(\beta + med\left(\frac{u'_i + u'_{i'}}{2}\right)\right).$$

$$\text{Hence,} \quad E(\widehat{\beta}'_A) = \beta, \quad \text{since} \quad E\left(med\left(\frac{u'_i + u'_{i'}}{2}\right)\right) = 0. \tag{26}$$

The variance of $\widehat{\beta}'_A$ is given by

$$V\left(\widehat{\beta}'_A\right) = \frac{1}{12m\overline{f}'^2} \tag{27}$$

where $\overline{f}' = \int f'^2\left(b'_i\right)\ db'_i$.

We observe that, both $\widehat{\beta}'_M$ and $\widehat{\beta}'_A$ are unbiased and consistent estimators of β.

Further, the model given in equation (19) is helpful in situations such as, in manufacturing process, while manufacturing semiconductor chip, analyzing maximum defect density observed at any stage of the process helps to ensure product quality, while minimizing the variation in product dimensions across different batches involve considering the minimum deviation. Moreover, calculating the average defect density or median product dimension provides insights into the typical performance of the manufacturing process, aiding in quality control and process optimization efforts. In stock market example, for instance, in assessing stock price volatility, identifying the maximum price fluctuation within a trading day helps gauge market turbulence, while analyzing the minimum price level provides insights into market support levels. Furthermore, calculating the average or median stock price over a specific period offers a more stable measure of market trends and investor sentiment, aiding in investment decision-making and risk management strategies.

6. Illustration

In this section, the proposed estimators are illustrated through an example. To fit simple linear regression model given in (1), the intercept parameter α is estimated using $\widehat{\alpha}_M = \widetilde{y} - \widehat{\beta}_M\widetilde{x}$ and $\widehat{\alpha}_A = \widetilde{y} - \widehat{\beta}_A\widetilde{x}$, where, $\widetilde{y}$ and $\widetilde{x}$ are median of y and x values respectively.

Example: The brain and weight data consisting of 62 samples due to Weisberg [22] is studied by Rousseeuw and Leroy [17] taking sample of 28 animals. Using this data that explains the brain weight (in grams) and the body weight (in kilograms) of 28 animals, we illustrate the proposed estimators. The body weight is represented by u and brain weight by v. The data is given by

Sr. No	Species	Body Weight (u)	Brain Weight (v)
1	Mountain beaver	1.35	8.1
2	Cow	465	423
3	Gray wolf	36.33	119.5
4	Goat	27.66	115
5	Guinea pig	1.04	5.5
6	Diplodocus	11700	50
7	Asian elephant	2547	4603
8	Donkey	187.1	419
9	Horse	521	655
10	Potar monkey	10	115
11	Cat	3.3	25.6
12	Giraffe	529	680
13	Gorilla	207	406
14	Human	62	1320
15	African elephant	6654	5712
16	Triceratops	9400	70
17	Rhesus monkey	6.8	179
18	Kangaroo	35	56
19	Hamster	0.12	1
20	Mouse	0.023	0.4
21	Rabbit	2.5	12.1
22	Sheep	55.5	175
23	Jaguar	100	157
24	Chimpanzee	52.16	440
25	Brachiosaurus	87000	154.5
26	Rat	0.28	1.9
27	Mole	0.122	3
28	Pig	192	180

To linearize the data, the observations on u and v are transformed to $x = log(u)$ and $y = log(v)$. We compute the values of $\widehat{\beta}_M$ and $\widehat{\beta}_A$ along with their variances in Exhibit 4.

Exhibit 4.: Computed values of proposed estimators and their variance under different distributions

Estimator	Value of estimator	Variance	Distribution of e_j
$\widehat{\beta}_M$	0.6216	$0.09657\sigma^2$	Uniform
		$0.10113\sigma^2$	Normal
		$0.25753\sigma^2$	Laplace
		$0.31771\lambda^2$	Cauchy
$\widehat{\beta}_A$	0.5668	$0.21251\sigma^2$	Uniform
		$0.19782\sigma^2$	Normal
		$0.64480\sigma^2$	Laplace
		$1.24295\lambda^2$	Cauchy

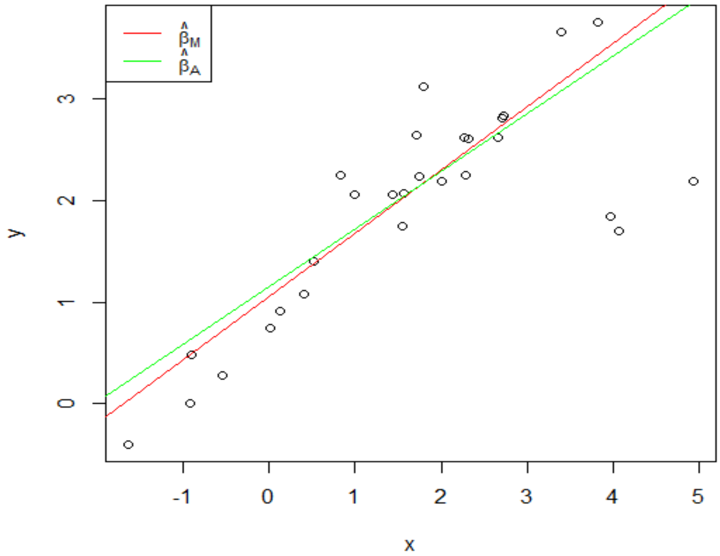

Figure 2.: The fitted regression lines using proposed estimators

From Exhibit 4 and Figure 2, it is observed that, the regression lines fitted using $\widehat{\beta}_M$ and $\widehat{\beta}_A$ are robust to outliers.

7. Conclusions

- The estimators based on median and HL type estimator using sample quasi ranges are proposed for slope parameter in SLR.
- The proposed estimators are unbiased, consistent for β and are robust in nature.
- As the estimators are developed for $n = 2m$, the middle observation is not considered when n is odd.
- The distributional properties of proposed estimators are derived and their performances are investigated when error follows various symmetric distributions.
- When $C_M = C_A$, $\widehat{\beta}_A$ performs better than $\widehat{\beta}_M$ under uniform, normal and Laplace distribution, whereas $\widehat{\beta}_M$ outperforms $\widehat{\beta}_A$ for Cauchy distribution in terms of their relative efficiency.
- For all the distributions under consideration, $\widehat{\beta}_M$ and $\widehat{\beta}_A$ outperform $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$. That is, the estimator obtained using quasi ranges are better than those based on half ranges.
- The proposed estimators are generalized to accommodate scenarios where multiple responses, $y_{jk}s$, are observed on single predictor, x_js. Additionally, the generalized regression model is adapted to situations where there is interest in functions of y_{jk}s.
- The simulation study indicates that, $\widehat{\beta}_M$ outperforms $\widehat{\beta}_A$, $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$ under Cauchy distribution, whereas, $\widehat{\beta}_A$ is better than $\widehat{\beta}_M$, $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$ under uniform, normal and Laplace distributions.
- When predictor variables have equal distance d, $\widehat{\beta}_M$ and $\widehat{\beta}_A$ have better per-

formance than $\widehat{\beta}_{TM}$ and $\widehat{\beta}_{TA}$. For increasing values of n and d, both estimators perform better than $\widehat{\beta}_{LS}$, $\widehat{\beta}_{Q}$, $\widehat{\beta}_{W}$, $\widehat{\beta}_{2C_2}$ and $\widehat{\beta}_{2D_2}$.
- As the proposed estimators are robust, they are helpful when data contains outliers.
- The estimators are also useful in the situations where predictor variables are evenly spaced as they outcompete all other estimators under consideration.

Appendix

Table 1.: V($\widehat{\beta}_M$) and V($\widehat{\beta}_A$) for various values of n

n	Uniform		Normal		Laplace		Cauchy	
	$V\left(\widehat{\beta}_M\right)$	$V\left(\widehat{\beta}_A\right)$	$V\left(\widehat{\beta}_M\right)$	$V\left(\widehat{\beta}_A\right)$	$V\left(\widehat{\beta}_M\right)$	$V\left(\widehat{\beta}_A\right)$	$V\left(\widehat{\beta}_M\right)$	$V\left(\widehat{\beta}_A\right)$
6	1.000000	0.750000	1.047198	0.698132	2.666667	2.275556	0.223251	0.297668
8	0.750000	0.562500	0.785398	0.523599	2.000000	1.706667	0.167438	0.223251
10	0.600000	0.450000	0.628319	0.418879	1.600000	1.365333	0.133951	0.178601
14	0.428571	0.321429	0.448799	0.299199	1.142857	0.975238	0.095679	0.127572
18	0.333333	0.250000	0.349066	0.232711	0.888889	0.758519	0.074417	0.099223
22	0.272727	0.204545	0.285599	0.190400	0.727273	0.620606	0.060887	0.081182
26	0.230769	0.173077	0.241661	0.161107	0.615385	0.525128	0.051520	0.068693
30	0.200000	0.150000	0.209440	0.139626	0.533333	0.455111	0.044650	0.059534
40	0.150000	0.112500	0.157080	0.104720	0.400000	0.341333	0.033488	0.044650
50	0.120000	0.090000	0.125664	0.083776	0.320000	0.273067	0.026790	0.035720
70	0.085714	0.064286	0.089760	0.059840	0.228571	0.195048	0.019136	0.025514
100	0.060000	0.045000	0.062832	0.041888	0.160000	0.136533	0.013395	0.017860
150	0.040000	0.030000	0.041888	0.027925	0.106667	0.091022	0.008930	0.011907
200	0.030000	0.022500	0.031416	0.020944	0.080000	0.068267	0.006698	0.008930
300	0.020000	0.015000	0.020944	0.013963	0.053333	0.045511	0.004465	0.005953
500	0.012000	0.009000	0.012566	0.008378	0.032000	0.027307	0.002679	0.003572

Table 2.: $\widehat{V}(\hat{\beta}_M)$, $\widehat{V}(\hat{\beta}_A)$, $\widehat{V}(\hat{\beta}_{TM})$ and $\widehat{V}(\hat{\beta}_{TA})$ under various error distributions

Uniform distribution					
σ	n	$\widehat{V}(\hat{\beta}_M)$	$\widehat{V}(\hat{\beta}_A)$	$\widehat{V}(\hat{\beta}_{TM})$	$\widehat{V}(\hat{\beta}_{TA})$
1	10	0.000156	0.000157	0.000260	0.000204
	20	0.000073	0.000066	0.000125	0.000096
	30	0.000043	0.000042	0.000080	0.000062
	50	0.000025	0.000024	0.000048	0.000037
	70	0.000017	0.000017	0.000034	0.000026
	100	0.000013	0.000011	0.000024	0.000018
5	10	0.003902	0.003930	0.006510	0.005093
	20	0.001831	0.001644	0.003124	0.002391
	30	0.001081	0.001059	0.002000	0.001562
	50	0.000630	0.000595	0.001200	0.000918
	70	0.000437	0.000421	0.000857	0.000649
	100	0.000334	0.000283	0.000600	0.000450
8	10	0.009990	0.010062	0.016667	0.013038
	20	0.004687	0.004208	0.007997	0.006122
	30	0.002768	0.002712	0.005120	0.003998
	50	0.001613	0.001523	0.003072	0.002351
	70	0.001120	0.001079	0.002194	0.001662
	100	0.000855	0.000726	0.001536	0.001152
10	10	0.015609	0.015722	0.026042	0.020371
	20	0.007324	0.006575	0.012495	0.009565
	30	0.004325	0.004237	0.008000	0.006247
	50	0.002520	0.002380	0.004800	0.003673
	70	0.001749	0.001686	0.003429	0.002597
	100	0.001337	0.001134	0.002400	0.001800

Normal distribution					
σ	n	$\widehat{V}(\hat{\beta}_M)$	$\widehat{V}(\hat{\beta}_A)$	$\widehat{V}(\hat{\beta}_{TM})$	$\widehat{V}(\hat{\beta}_{TA})$
1	10	0.000163	0.000146	0.000273	0.000190
	20	0.000074	0.000060	0.000131	0.000089
	30	0.000044	0.000039	0.000087	0.000058
	50	0.000026	0.000022	0.000050	0.000034
	70	0.000018	0.000016	0.000036	0.000024
	100	0.000014	0.000011	0.000025	0.000017
5	10	0.004086	0.003659	0.006818	0.004741
	20	0.001917	0.001504	0.003271	0.002226
	30	0.001100	0.000978	0.002181	0.001439
	50	0.000660	0.000554	0.001257	0.000855
	70	0.000458	0.000392	0.000898	0.000604
	100	0.000340	0.000264	0.000628	0.000419
8	10	0.010461	0.009366	0.017453	0.012136
	20	0.004833	0.003917	0.008374	0.005698
	30	0.002815	0.002503	0.005583	0.003684
	50	0.001689	0.001418	0.003217	0.002188
	70	0.001207	0.001013	0.002298	0.001532
	100	0.000870	0.000697	0.001608	0.001072
10	10	0.016345	0.014635	0.027271	0.018962
	20	0.007551	0.006120	0.013085	0.008904
	30	0.004529	0.003911	0.008378	0.005815
	50	0.002565	0.002215	0.005027	0.003419
	70	0.001832	0.001582	0.003590	0.002418
	100	0.001359	0.001090	0.002513	0.001676
Laplace distribution					
σ	n	$\widehat{V}(\hat{\beta}_M)$	$\widehat{V}(\hat{\beta}_A)$	$\widehat{V}(\hat{\beta}_{TM})$	$\widehat{V}(\hat{\beta}_{TA})$
1	10	0.0004300	0.0004770	0.0006944	0.0006314
	20	0.0001984	0.0002065	0.0003332	0.0002902
	30	0.0001153	0.0001286	0.0002221	0.0001876
	50	0.0000672	0.0000734	0.0001280	0.0001114
	70	0.0000466	0.0000520	0.0000914	0.0000788
	100	0.0000356	0.0000355	0.0000640	0.0000546
5	10	0.0000172	0.0000191	0.0000278	0.0000253
	20	0.0000079	0.0000083	0.0000133	0.0000116
	30	0.0000046	0.0000051	0.0000089	0.0000075
	50	0.0000027	0.0000029	0.0000051	0.0000045
	70	0.0000019	0.0000021	0.0000037	0.0000032
	100	0.0000014	0.0000014	0.0000026	0.0000022
8	10	0.0000067	0.0000075	0.0000109	0.0000099
	20	0.0000031	0.0000032	0.0000052	0.0000045
	30	0.0000018	0.0000020	0.0000035	0.0000029
	50	0.0000011	0.0000011	0.0000020	0.0000017
	70	0.0000007	0.0000008	0.0000014	0.0000012
	100	0.0000006	0.0000006	0.0000010	0.0000009
10	10	0.0000043	0.0000048	0.0000069	0.0000063
	20	0.0000020	0.0000021	0.0000033	0.0000029
	30	0.0000012	0.0000013	0.0000022	0.0000019
	50	0.0000007	0.0000007	0.0000013	0.0000011
	70	0.0000005	0.0000005	0.0000009	0.0000008
	100	0.0000004	0.0000004	0.0000006	0.0000005
Cauchy distribution					
λ	n	$\widehat{V}(\hat{\beta}_M)$	$\widehat{V}(\hat{\beta}_A)$	$\widehat{V}(\hat{\beta}_{TM})$	$\widehat{V}(\hat{\beta}_{TA})$
0.2605	10	0.000037	0.000066	0.000061	0.000083
	20	0.000018	0.000028	0.000028	0.000039
	30	0.000010	0.000018	0.000019	0.000025
	50	0.000006	0.000010	0.000011	0.000015
	70	0.000004	0.000007	0.000008	0.000010
	100	0.000003	0.000005	0.000005	0.000007
1	10	0.000548	0.000973	0.000894	0.001217
	20	0.000261	0.000412	0.000420	0.000565
	30	0.000151	0.000263	0.000274	0.000365

	50	0.000083	0.000149	0.000158	0.000215
	70	0.000059	0.000104	0.000113	0.000152
	100	0.000044	0.000071	0.000079	0.000105
5	10	0.013708	0.024333	0.022340	0.030430
	20	0.006524	0.010306	0.010490	0.014133
	30	0.003776	0.006576	0.006851	0.009135
	50	0.002073	0.003717	0.003948	0.005371
	70	0.001481	0.002589	0.002820	0.003798
	100	0.001099	0.001768	0.001974	0.002632
8	10	0.035092	0.062293	0.057189	0.077901
	20	0.016701	0.026383	0.026853	0.036179
	30	0.009667	0.016835	0.017539	0.023385
	50	0.005307	0.009516	0.010106	0.013749
	70	0.003791	0.006629	0.007219	0.009722
	100	0.002814	0.004527	0.005053	0.006738
10	10	0.054831	0.097333	0.089358	0.121720
	20	0.026095	0.041223	0.041958	0.056530
	30	0.015105	0.026305	0.027404	0.036539
	50	0.008292	0.014868	0.015791	0.021483
	70	0.005923	0.010358	0.011280	0.015191
	100	0.004397	0.007073	0.007896	0.010528

Table 3.: $\widehat{V}(\hat{\beta}_{LS})$, $\widehat{V}(\hat{\beta}_{Q})$, $\widehat{V}(\hat{\beta}_{W})$, $\widehat{V}(\hat{\beta}_{2C_2})$ and $\widehat{V}(\hat{\beta}_{2D_2})$ under normal error distribution

σ	n	$\widehat{V}(\hat{\beta}_{LS})$	$\widehat{V}(\hat{\beta}_{Q})$	$\widehat{V}(\hat{\beta}_{W})$	$\widehat{V}(\hat{\beta}_{2C_2})$	$\widehat{V}(\hat{\beta}_{2D_2})$
1	10	0.000133	0.000189	0.000142	0.000182	0.000138
	20	0.000063	0.000086	0.000065	0.000071	0.000064
	30	0.000041	0.000056	0.000042	0.000043	0.000041
	50	0.000024	0.000033	0.000024	0.000025	0.000024
	70	0.000017	0.000023	0.000017	0.000017	0.000017
	100	0.000012	0.000016	0.000012	0.000012	0.000012
5	10	0.003341	0.004726	0.003559	0.004543	0.003465
	20	0.001566	0.002161	0.001612	0.001763	0.001594
	30	0.001026	0.001400	0.001043	0.001081	0.001037
	50	0.000607	0.000816	0.000611	0.000617	0.000609
	70	0.000430	0.000575	0.000431	0.000433	0.000431
	100	0.000300	0.000400	0.000300	0.000300	0.000300
8	10	0.008457	0.011994	0.009029	0.011652	0.008789
	20	0.004023	0.005556	0.004143	0.004525	0.004098
	30	0.002623	0.003573	0.002668	0.002760	0.002653
	50	0.001551	0.002085	0.001563	0.001579	0.001560
	70	0.001100	0.001473	0.001103	0.001107	0.001102
	100	0.000768	0.001024	0.000768	0.000768	0.000768
10	10	0.013281	0.018904	0.014172	0.018171	0.013795
	20	0.006273	0.008644	0.006462	0.007076	0.006402
	30	0.004099	0.005584	0.004173	0.004311	0.004147
	50	0.002425	0.003262	0.002442	0.002467	0.002436
	70	0.001722	0.002307	0.001728	0.001734	0.001726
	100	0.001200	0.001600	0.001200	0.001201	0.001200

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